

# Thermal quantum discord in the anisotropic Heisenberg $XYZ$ model with the Dzyaloshinskii-Moriya interaction

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The various thermal quantum correlations as well as the effect of intrinsic decoherence on the correlations is studied in a two-qubit Heisenberg  $XYZ$  spin chain with the Dzyaloshinski-Moriya ( $D - M$ ) interaction. The quantum entanglement ( $C$ ) of the system is also discussed and compared with the classical correlation ( $CC$ ) and quantum discord ( $QD$ ). We show situation where both  $QD$  and  $CC$  decreases asymptotically to zero with temperature  $T$  while entanglement decreases to zero at the point of critical temperature. situation where  $C$ ,  $CC$  and  $QD$  increases with certain tunable parameter such as  $D_z$ . We also show that tunable parameter  $D_z$  may plays a constructive role to the  $C$ ,  $CC$  and  $QD$  in thermal equilibrium.

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## I. INTRODUCTION

A generic bipartite quantum state mathematically represented by a density operator, is a hybrid object with both classical and quantum characteristics. It can encode classical as well as quantum correlations by means of superposition, entanglement, and mixing. How to distinguish these two kinds of correlations is of basic importance and interest in quantum information theory.

Entanglement is a kind of quantum nonlocal correlation and has been deeply studied in the past years[1–3], nevertheless, the quantum discord measures quantum correlations of a more general type of quantum correlation, and there exists separable mixed state having nonzero quantum discord[4].  $QD$  is built on the fact that two classical equivalent ways of defining the mutual information turn out to be in equivalent in the quantum domain. In addition to its conceptual role, some recent results [5] suggest that  $QD$  and entanglement may be responsible for the efficiency of a mixed state based quantum computer.

Recently, the  $QD$  has been intensively investigated in the literature both theoretically [8–31] and experimentally [6–32]. Generally, it is somewhat difficult to calculate  $QD$  and the analytical solutions can hardly be obtained except for some particular cases, such as the so-called X states [11]. Some researches show that  $QD$ , concurrence ( $C$ ) and classical correlation ( $CC$ ) are respectively independent measures of correlations with no simple relative ordering and  $QD$  is more practical than entanglement [6]. Quite recently, B.Dakic et al [25] have introduced an easily analytically computable quantity, geometric measure of discord ( $GMD$ ), and given a necessary and sufficient condition for the existence of nonzero  $QD$  for any dimensional bipartite states. Moreover, the dynamical behavior of  $QD$  in terms of decoherence [28–35] in both Markovian [11] and Non-Markovian [13, 36, 37] cases is also taken into account.

In previous studies, the quantum discord of a two qubit one-dimensional  $XYZ$  Heisenberg chain in thermal equilibrium

has been studied [38] where many unexpected ways different from the thermal entanglement have been shown. For the Heisenberg model, many properties have been studied, however, the Dzyaloshinski-Moriya ( $D - M$ ) anisotropic antisymmetric interaction has rarely been considered [39, 40] which arises from the weak inter-molecular interactions and describes an interaction of extended super exchange mechanism by considering a term arising naturally from the perturbation theory due to the spin-orbit coupling. It has been shown that the D-M interaction may play an important role in the entanglement of the bipartite system and the three-particle system [41–45]. Thus, considering the quantum discord in a two qubit system with the D-M interaction is also interesting. In this paper, we will study the quantum discord in the anisotropic Heisenberg  $XYZ$  model with the Dzyaloshinskii-Moriya interaction, and discuss how the  $D - M$  interaction affects the various correlation in such system.

The rest of this paper is organized as follows. We give the analytic solution of the system in section 1, then discuss the effect  $D - M$  interaction in thermalized and Intrinsic decoherence case on various correlation in section 2 and section 3 and give the summary in section 4 finally.

## II. THE THERMALIZED HEISENBERG SYSTEM

We consider the anisotropic  $XYZ$  Heisenberg model with the anisotropic, antisymmetric  $D - M$  interaction. The  $D - M$  anisotropic, anti-symmetric interaction arises from spin-orbit coupling, which can be described as  $D \bullet (\sigma_1 \times \sigma_2)$ . In our study, we consider only the case of D-M interaction along the  $z$  direction  $D_z(\sigma_1^x \sigma_2^y - \sigma_1^y \sigma_2^x)$ , then the Hamiltonian of such a model can be expressed as

$$H = \frac{1}{2}[J_x \sigma_1^x \sigma_2^x + J_y \sigma_1^y \sigma_2^y + J_z \sigma_1^z \sigma_2^z + D_z (\sigma_1^x \sigma_2^y - \sigma_1^y \sigma_2^x)] \quad (1)$$

where  $J_x$ ,  $J_y$  and  $J_z$  are the coupling constants;  $\sigma_i^x$ ,  $\sigma_i^y$  and  $\sigma_i^z$  are the Pauli operators acting on qubit  $i$  ( $i = 1, 2$ ). In the standard basis  $|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle$ , the Hamiltonian can be

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expressed in the following matrix form

$$H = \frac{1}{2} \begin{pmatrix} J_z & 0 & 0 & J_x - J_y \\ 0 & -J_z & \beta & 0 \\ 0 & \beta^* & -J_z & 0 \\ J_x - J_y & 0 & 0 & J_z \end{pmatrix} \quad (2)$$

Where  $\beta = J_x + J_y + 2iD_z$ . We are working in units, so that all parameters are dimensionless. The state of a typical solid-state system at thermal equilibrium in temperature  $T$  (canonical ensemble) is  $\rho(T) = \frac{\exp(-\frac{H}{KT})}{Z}$ , with  $Z = \text{tr}[\exp(-\frac{H}{KT})]$  the partition function and  $K$  is the Boltzmann constant. Usually we work with natural unit system  $\hbar = K = 1$  for simplicity and henceforth. This density matrix can be worked out to

$$\rho(T) = \frac{1}{Z} \begin{pmatrix} A_{11} & 0 & 0 & A_{41} \\ 0 & B_{22} & B_{23}^\dagger & 0 \\ 0 & B_{23} & B_{22} & 0 \\ A_{41} & 0 & 0 & A_{11} \end{pmatrix} \quad (3)$$

where the elements of the matrix have been defined as

$$A_{11} = \frac{1}{2} [\exp(-\frac{J_x + J_y + J_z}{2T}) (\exp(\frac{J_x}{T}) + \exp(\frac{J_y}{T}))],$$

$$A_{41} = \frac{1}{2} [\exp(-\frac{J_x + J_y + J_z}{2T}) (-\exp(\frac{J_x}{T}) + \exp(\frac{J_y}{T}))],$$

$$B_{22} = \frac{\exp(\frac{J_z}{2T}) \cosh[\frac{\sqrt{(J_x + J_y)^2 + 4D_z^2}}{2T}]}{Z},$$

$$B_{23} = \frac{(J_x + J_y - 2iD_z) \exp(\frac{J_z}{2T}) \sinh[\frac{\sqrt{(J_x + J_y)^2 + 4D_z^2}}{2T}]}{\sqrt{(J_x + J_y)^2 + 4D_z^2} Z},$$

and

$$Z = \exp(-\frac{J_x + J_y + J_z}{2T}) (\exp(\frac{J_x}{T}) + \exp(\frac{J_y}{T})) + 2\exp(\frac{J_z}{2T}) \cosh[\frac{\sqrt{(J_x + J_y)^2 + 4D_z^2}}{2T}].$$

### III. EFFECTS OF DM INTERACTION ON VARIOUS THERMAL CORRELATIONS

Firstly we give a brief overview of various correlation measures. Given a bipartite quantum state  $\rho_{AB}$  in a composite Hilbert space  $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$ , the concurrence [4–6] as an indicator for entanglement between the two qubits is

$$C(\rho_{AB}) = \max\{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0\}, \quad (4)$$

where  $\lambda_i (i = 1, 2, 3, 4)$  are the square roots of the eigenvalues of the "spin-flipped" density operator  $R = \rho \tilde{\rho} = \rho(\sigma_1^y \otimes \sigma_2^y) \rho^* (\sigma_1^y \otimes \sigma_2^y)$  in descending order.  $\sigma_y$  is the Pauli matrix and  $\rho^*$  denotes the complex conjugation of the matrix  $\rho$  in the standard basis  $|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle$ . Let us now recalling the original definition of QD, in classical information theory the total correlations in a bipartite quantum system ( $A$ ) and

( $B$ ) are measured by the quantum mutual information defined as

$$\mathcal{I}(\rho_A; \rho_B) = S(\rho_A) + S(\rho_B) - S(\rho_{AB}), \quad (5)$$

where  $\rho_{A(B)} = \text{Tr}_{B(A)}(\rho_{AB})$  is the reduced density matrix of the subsystem  $A(B)$  by tracing out the subsystem  $B(A)$ . The quantum generalization of the conditional entropy is not the simply replacement of Shannon entropy with von Neumann entropy, but through the process of projective measurement on the subsystem  $B$  by a set of complete projectors  $B_k$ , with the outcomes labeled by  $k$ , then the conditional density matrix  $\rho_k$  becomes

$$\rho_k = \frac{1}{p_k} (\mathbb{I}_A \otimes B_k) \rho (\mathbb{I}_A \otimes B_k), \quad (6)$$

which is the locally post-measurement state of the subsystem  $B$  after obtaining the outcome  $k$  on the subsystem  $A$  with the probability

$$p_k = \text{Tr}[(\mathbb{I}_A \otimes B_k) \rho (\mathbb{I}_A \otimes B_k)], \quad (7)$$

where  $\mathbb{I}_A$  is the identity operator on the subsystem  $A$ . The projectors  $B_k$  can be parameterized as  $B_k = V|k\rangle\langle k|V^\dagger$ ,  $k = 0, 1$  and the transform matrix  $V \in U(2)$  [8] is

$$V = \begin{pmatrix} \cos \theta & e^{-i\phi} \sin \theta \\ e^{i\phi} \sin \theta & -\cos \theta \end{pmatrix}. \quad (8)$$

Then the conditional von Neumann entropy (quantum conditional entropy) and quantum extension of the mutual information can be defined as [4]

$$S(\rho|\{B_k\}) = \sum_k p_k S(\rho_k), \quad (9)$$

Following the definition of the CC in Ref.[4]

$$CC(\rho_{AB}) = \sup_{\{B_k\}} \{S(\rho_A(t)) - S(\rho_{AB}(t)|\{B_k\})\}, \quad (10)$$

then  $QD$  defined by the difference between the quantum mutual information  $\mathcal{I}(\rho_{AB})$  and the  $CC(\rho_{AB})$  is given by  $QD(\rho_{AB}) = \mathcal{I}(\rho_{AB}) - CC(\rho_{AB})$ . If we denote  $S_{\min}(\rho_{AB}) = \min_{\{B_k\}} S(\rho_{AB}|\{B_k\})$ , Then a variant expression of  $CC$  and  $QD$  in Ref.[13]

$$CC(\rho_{AB}) = S(\rho_A) - \min_{\{B_k\}} S(\rho_{AB}|\{B_k\}), \quad (11)$$

$$QD(\rho_{AB}) = S(\rho_B) - S(\rho_{AB}) + S_{\min}(\rho_{AB}), \quad (12)$$

Figure 1.a Shows that In temperature  $T$  smaller, along with the increase of also become larger  $C(T)$ . In figure 1.b and figure 1.c, in the limited temperature, along with the increase of  $CC(T)$  and  $QD(T)$  as increases until the critical value. Temperature  $T$  larger than 1.8 later  $CC(T)$  and  $QD(T)$  unchanged that is equal to zero.

We concluded that  $D_z$  turn out to be the most efficient parameters in increasing various correlations such as  $C$ ,  $CC$  and  $QD$  in the finite temperature.

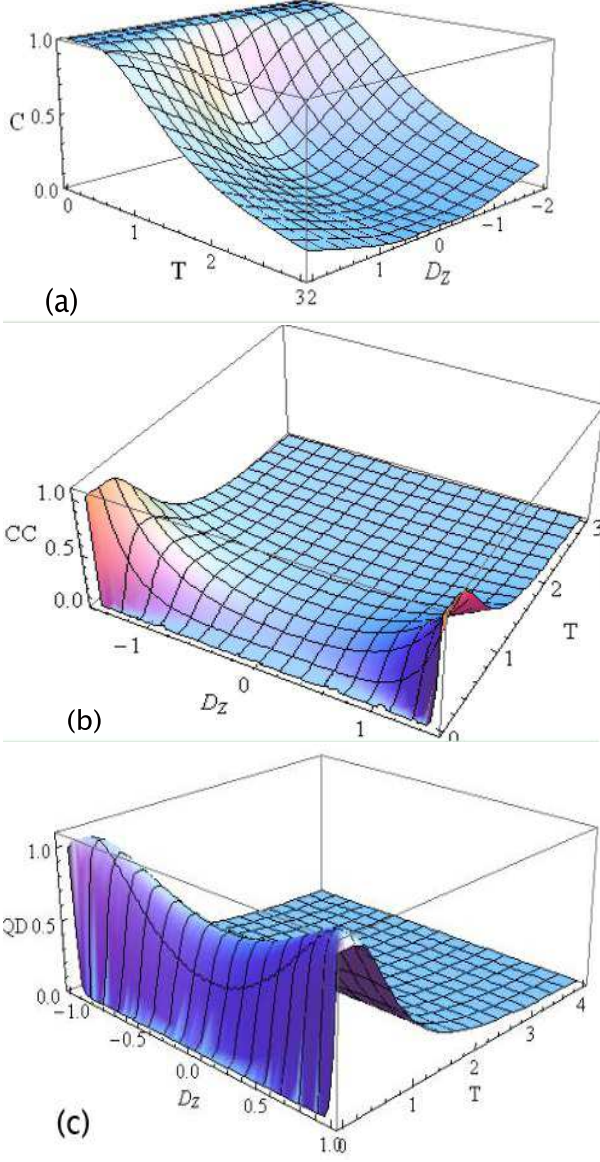


FIG. 1: (Color online) As three different quantifiers of correlations. The Concurrence (FIG. 1.a with), The Classical correlation (FIG. 1.b with) and quantum Discord (FIG. 1.c with) as a function of the absolute temperature  $T$  and the  $D - M$  interaction, The above three plots in the  $XYZ$  model without external magnetic field.

#### IV. INTRINSIC DECOHERENCE OF VARIOUS CORRELATIONS

We consider the influence of intrinsic decoherence, proposed by Milburn [47] with the assumption that a system does not evolve continuously under unitary transformation for sufficiently short time steps, on various correlations. The master equation describing the intrinsic decoherence can be formu-

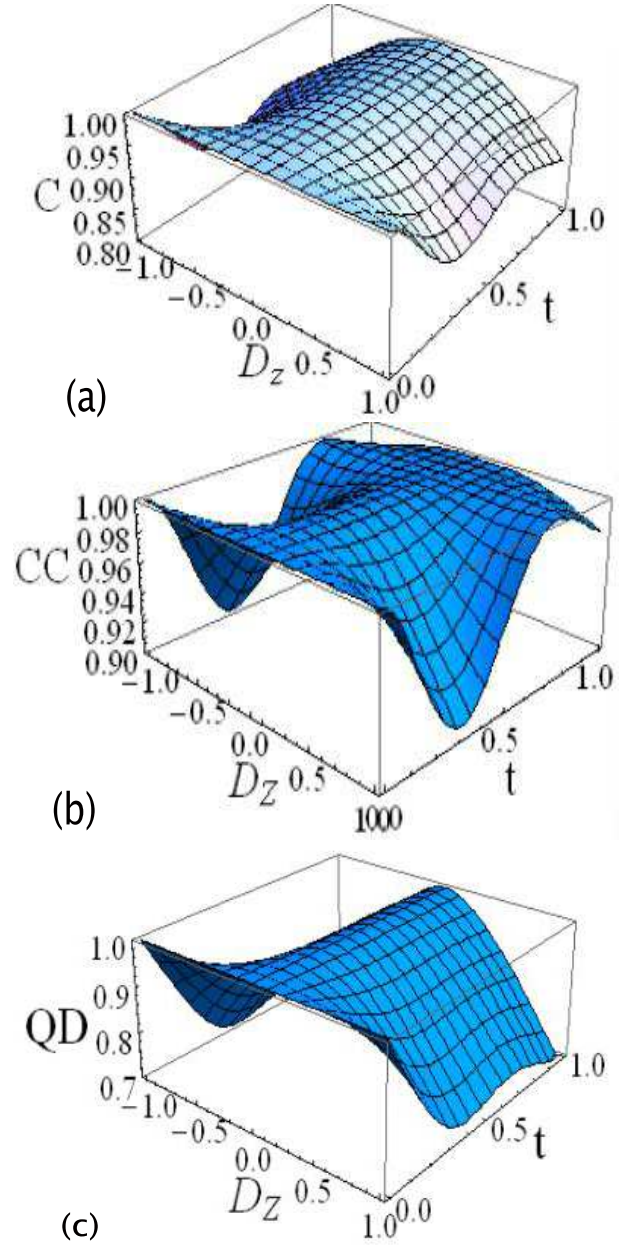


FIG. 2: (Color online) as three different quantifiers of correlations. The concurrence (FIG. 2.a), the classical correlation (FIG. 2.b) and quantum discord (FIG. 2.c) as a function of the decoherence time  $t$  and the  $D - M$  interaction  $D_z$ , The above three plots in the  $XYZ$  model without external magnetic field. Here  $J_x = 3$ ,  $J_y = 0.6$ ,  $D_z = 1$ ,  $\gamma = 0.1$ .

lated as

$$\frac{d\rho(t)}{dt} = -i[H, \rho] - \frac{1}{2\gamma}[H, [H, \rho(t)]], \quad (13)$$

where  $\gamma$  is the phase decoherence rate. Schrodinger's equation is recovered in the limit  $\gamma \rightarrow 0$ . The formal solution of the



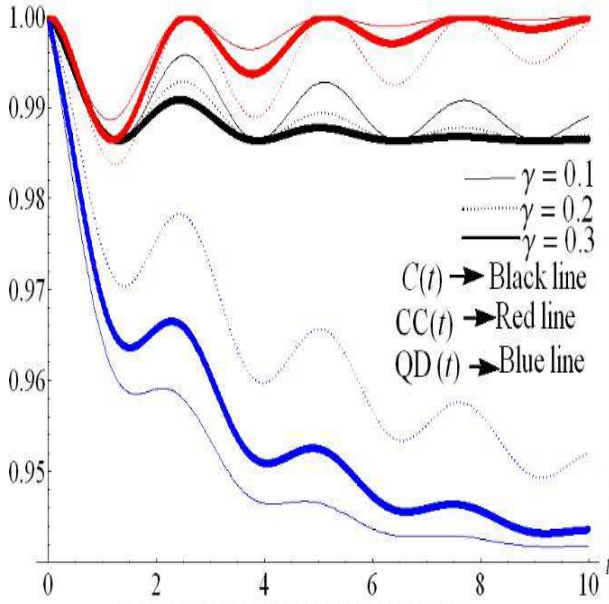


FIG. 3: (Color online) The various quantum correlation  $C$ ,  $QD$ ,  $CC$  versus time  $t$  at a given  $\gamma = 0.1, 0.2, 0.3$ . Here  $C$ ,  $QD$ ,  $CC$  corresponds respectively to the black, blue, red line.

above master equation can be expressed as[48]

$$\rho(t) = \sum_{k=0}^{\infty} \frac{t^k}{k!} M^k(t) \rho(0) M^{\dagger k}(t), \quad (14)$$

where  $\rho(0)$  is the density operator of the initial system and  $M^k(t)$  is defined by

$$M^k(t) = H^k \exp(-iHt) \exp\left(-\frac{t}{2\gamma} H^2\right). \quad (15)$$

Insert the complete relation  $\sum_n |\psi_n\rangle\langle\psi_n|$  of the energy eigenstate into master equation [49], one can write the explicit expression of the density matrix of the states as

$$\begin{aligned} \rho(t) = \sum_{mn} \exp\left[-\frac{\gamma t}{2}(E_m - E_n)^2 - i(E_m - E_n)t\right] \\ \times \langle\psi_m|\rho(0)|\psi_n\rangle |\psi_m\rangle\langle\psi_n|, \end{aligned} \quad (16)$$

We assume that the system is initially prepared in the Bell state  $|\Psi_1(0)\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$  From equation (16), the time

evolution of for this initial state can be obtained as

$$\rho(t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (17)$$

where the elements of the matrix have been defined as

$$\begin{aligned} \rho_{22} &= \frac{1}{2} + \frac{D_z \exp(-\frac{1}{2}t\gamma\mu^2) \sin[t\mu]}{\mu^2}; \\ \rho_{23} &= \frac{J_x + J_y - 2iD_z \exp(-\frac{1}{2}t\gamma\mu^2) \cos[t\mu]}{2(J_x + J_y - 2iD_z)}; \\ \rho_{32} &= \frac{J_x + J_y + 2iD_z \exp(-\frac{1}{2}t\gamma\mu^2) \cos[t\mu]}{2(J_x + J_y + 2iD_z)}; \\ \rho_{33} &= \frac{1}{2} - \frac{D_z \exp(-\frac{1}{2}t\gamma\mu^2) \sin[t\mu]}{\mu^2} \end{aligned}$$

and where

$$\mu = \sqrt{J_x^2 + 2J_x J_y + J_y^2 + 4D_z^2}$$

In figure.2, we found that as  $D_z$  and  $t$  increase the  $C(t)$ ,  $CC(t)$  and  $QD(t)$  are down, but as  $D_z$  take a small value. the  $C(t)$ ,  $CC(t)$  and  $QD(t)$  drop a slow speed, especially  $CC(t)$  minimum values more than 0.9.

In figure 3, we found that when the quantum correlation  $CC(t)$  is little change and along with the increase of also become smaller.  $QD(t)$  take a maximum value in  $\gamma = 0.2$ , take a minimum value in  $\gamma = 0.1$ . But  $CC(t)$  is just the opposite, when  $\gamma = 0.1$  take a maximum value,  $\gamma = 0.2$  take a minimum value.

## V. CONCLUSION

In conclusion ,we have studied the quantum discord in an anisotropic two qubits Heisenberg  $XYZ$  chain with the presence of  $D - M$  interaction and relation between various correlation and the  $D - M$  interaction .Our results show that the quantum discord can describe more information about classical correlation and quantum correlation than quantum entanglement and more properties can be revealed by quantum discord. Numerical results indicate that  $D-M$  interaction  $D_z$  plays a constructive role in thermal equilibrium. But  $D_z$  becomes to be destructive presence intrinsic decoherence.

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